

# REPORT DOCUMENTATION PAGE

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14. ABSTRACT The research performed during this AFOSR grant has extended the basic discrete DMOC (Discrete Mechanics and Optimal Control) framework to derive algorithms for general nonholonomic multi-body systems, with applications to a variety of problems including space mission design. A computational framework was developed which can automatically construct integration and optimization schemes by providing a high-level description of the mechanical system in terms of its physical layout, inertial properties, constraints, actuation, and external influences. This last year of the grant was focused on optimization problems with non-holonomic constraints as well as continued applications to space mission design. Besides describing the latest results and publications obtained over the past 12 months, we also describe the next steps that we expect to explore now that the work of this grant is over.				
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## Annual Performance Report, 2011

Grant No: FA9550-08-1-0173

*Geometric Computational Mechanics and Optimal Control  
(PIs: Marsden/Desbrun)*

The last year of this grant continued the development and numerical implementation of DMOC (Discrete Mechanics and Optimal Control) with applications to a variety of optimization problems. Professor J.E. Marsden, PI on this grant, unfortunately passed away this past September; Prof. Desbrun, who has collaborated with Marsden and his postdocs for the past six years on DMOC-related projects, has taken over the task of completing this research effort.

This year was focused on optimization problems with non-holonomic constraints as well as continued applications to space mission design. The method builds on what is by now well developed and successful techniques for the theoretical and numerical implementation of discrete mechanics done over the last decade. DMOC combines these discrete mechanics techniques with direct methods for optimal control, such as SQP (sequential quadratic programming) and related methods. Further methods that depend on global optimization problems are in development and preliminary versions of these results, many of which were obtained with the current postdoctoral fellow Marin Kobilarov, were reported in the recent annual meeting.

This grant's achievements were done not only with graduate students, but also with the recent past AFOSR (partially) supported postdoc, Sigrid Leyendecker, a recent winner of an Emmy Noether 5 year research award to start her group in Germany and Sina Ober-Blöbaum, now a junior professor at Paderborn. Our collaborations on DMOC are continuing, for example with Ober-Blöbaum on multiscale and receding horizon versions of DMOC with applications to circuits, and with Leyendecker on 3D multibody simulations and design of dynamics.

The concrete progress that was made since the last reporting period include:

## **Basic Optimal Control Theory and Applications**

This line of work is based on variational methods for time evolution for constrained systems, which showed a marked improvement over existing methods. The techniques devised in previous years of this grant have also been extended, both time evolution and optimal control problems, to non-holonomic constraints and Lie groups:

M. Kobilarov, J. Marsden, *Discrete Geometric Optimal Control on Lie Groups*, IEEE Transactions on Robotics and Automation, to appear, 2011.

M. Kobilarov, D. Martín de Diego, S. Ferraro, *Simulating Nonholonomic Dynamics*. Boletin de la Sociedad Espanola de Matematica Aplicada (SeMA), 50, 2010, pp 61-81.

K. Flaßkamp, S. Ober-Blöbaum, M. Kobilarov, *Solving optimal control problems by using inherent dynamical properties*, Proceedings in Applied Mathematics and Mechanics, 2010, vol. 10-1, pp. 577–578

S. Dubljevic, M. Kobilarov, and James Ng, *Discrete Mechanics Optimal Control (DMOC) and Model Predictive Control (MPC) Synthesis for Reaction-Diffusion Process System with Moving Actuator*, American Control Conference (ACC), 2010, pp. 5694-5701.

## **Applications to Space Mission Design.**

The optimization of spacecraft trajectories has been a natural application area for DMOC. We published a paper this year building upon last year's spacecraft trajectory optimization paper with DMOC to include refinement strategies for faster convergence:

A. Moore, Sina Ober-Blöbaum, and J.E. Marsden, *Mesh Refinement Strategies for Spacecraft Trajectory Optimization using Discrete Mechanics and Optimal Control*, AAS 11-127, 2011

## **Conclusions & Next Steps**

The research performed during this AFOSR grant extended the basic discrete DMOC framework to derive algorithms for general nonholonomic

multi-body systems. A computational framework was developed which can automatically construct integration and optimization schemes by providing a high-level description of the mechanical system in terms of its physical layout, inertial properties, constraints, actuation, and external influences. The main focus was then to extract and factor out as much structure as possible to obtain algorithms with lowest dimension that are amenable to controllability and convergence analysis, i.e., determining whether the system can reach a desired state and achieve it in an optimal way.

In this context a novel optimal control method was developed to exploit the structure of optimal trajectories. As a result, instead of relying on a black-box optimization package, optimal control trajectories are derived through a discrete optimality principle similar to the construction of higher order geodesics in the continuous setting.

Consequently, globally optimal methods for computing optimal trajectories for vehicles with complex dynamics were developed. The basic idea is to use motions optimized using discrete optimal control in order to generate near-optimal trajectories which probabilistically explore the state space searching for the optimal path. The set of all such trajectories are approximated using a graph with nodes corresponding to randomly sampled states. The methodology is designed to converge to an optimal solution as the number of nodes is increased. The approach was successfully applied to a variety of systems in simulation as well as to a real vehicle -- an unmanned helicopter flying in an urban terrain. This promising research direction will now be further studied in the context of future grants.